ECE 344
Microwave Fundamentals

Lecture 05:
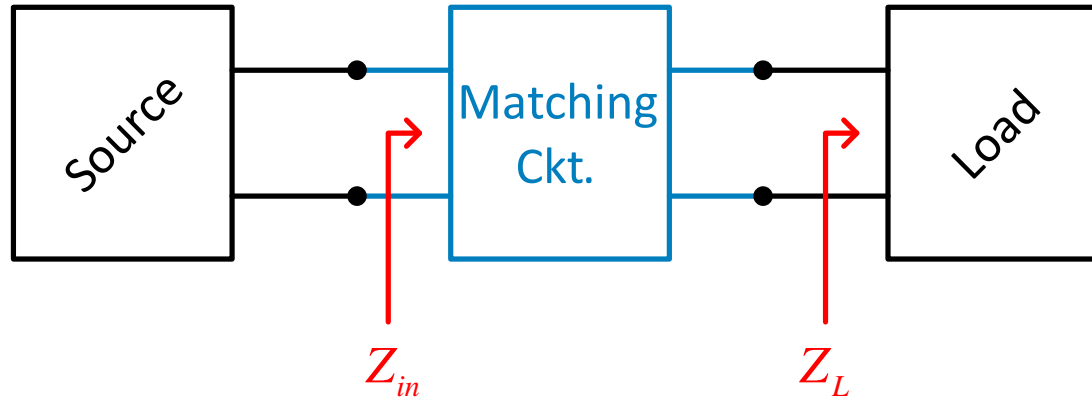## Impedance Matching

Prepared By

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Impedance matching or tuning is important for the following reasons:

- Maximum power is delivered when the load is matched to the line (assuming the generator is matched), and power loss in the feed line is minimized.

- Impedance matching sensitive receiver components (antenna, low-noise amplifier, etc.) improves the signal-to-noise ratio of the system.

- Impedance matching in a power distribution network (such as an antenna array feed network) will reduce amplitude and phase errors.

Important factors in the selection of a particular matching network

- Complexity --- As with most engineering solutions, the simplest design that satisfies the required specifications is generally the most preferable. A simpler matching network is usually cheaper, more reliable, and less lossy than a more complex design.

- Bandwidth --- Any type of matching network can ideally give a perfect match (zero reflection) at a single frequency. In many applications, however, it is desirable to match a load over a band of frequencies. There are several ways of doing this with, of course, a corresponding increase in complexity.

- Implementation --- Depending on the type of transmission line or waveguide being used, one type of matching network may be preferable compared to another. For example, tuning stubs are much easier to implement in waveguide than the multi-section quarter-wave transformers.

- Adjustability --- In some applications the matching network may require adjustment to match a variable load impedance. Some types of matching networks are more amenable than others in the regard.

Impedance Matching

Impedance matching is used to:

- Maximize power from source to load
- Minimize reflections
- Set terminating condition

Considerations:

- Complexity
- Implementation
- Bandwidth
- Adjustability

We have $Z_L = R_L + jX_L$

We want $Z_{in} = R_{in} + jX_{in}$

$\Downarrow$ Two constraints

Matching circuit typically requires at least 2 degrees of freedom.

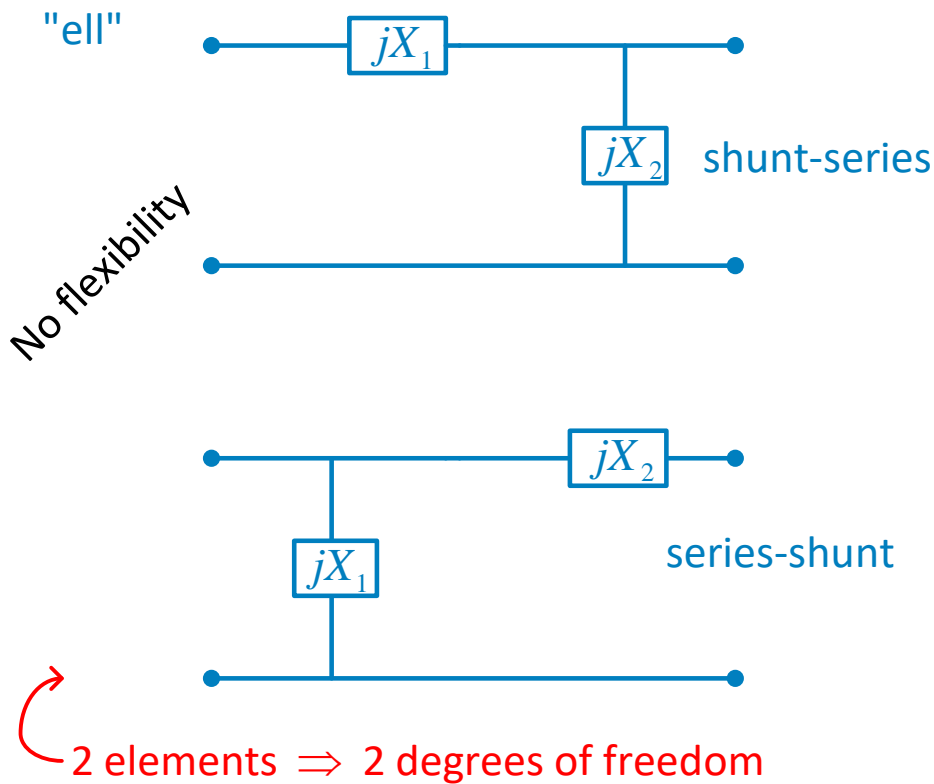# Impedance Matching (cont.)

Source — Matching Ckt. — Load
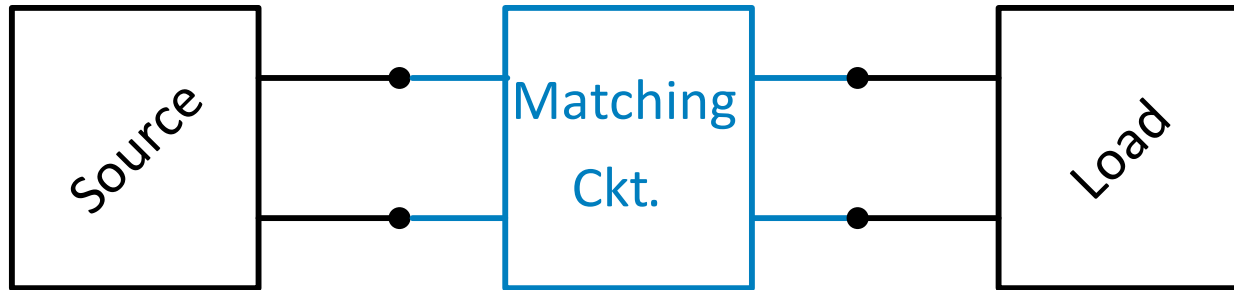
We will consider:

1) Lumped element matching circuits

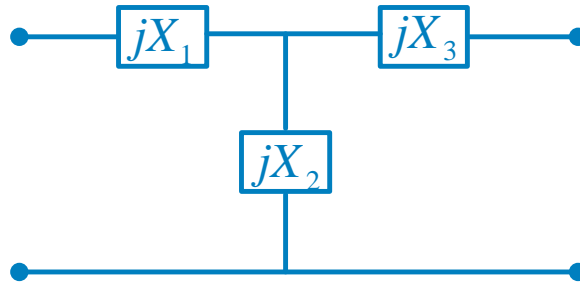2) Transmission line matching circuits

3) Quarter-wave Impedance transformers

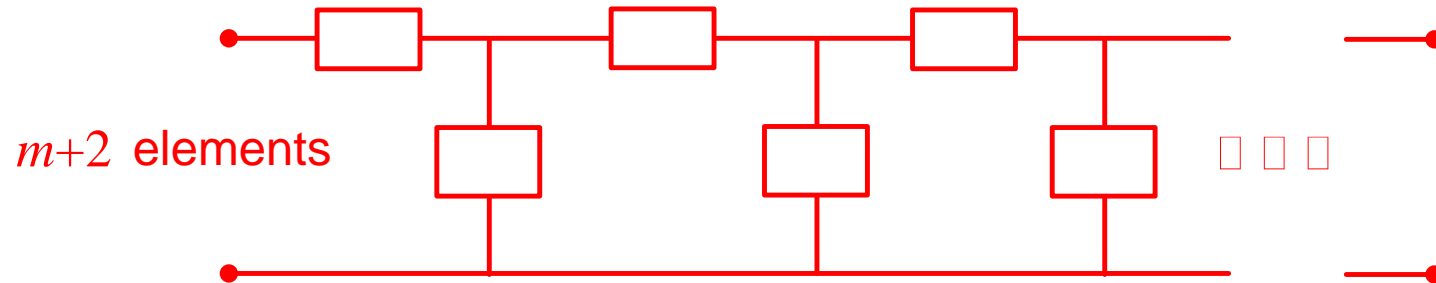# Lumped-Element Matching Circuits

Examples

"ell"

jX_1

jX_2 shunt-series

No flexibility

jX_2

jX_1 series-shunt

2 elements $\Rightarrow$ 2 degrees of freedom

"pi"

jX_2

jX_1 jX_3

One extra degree of freedom
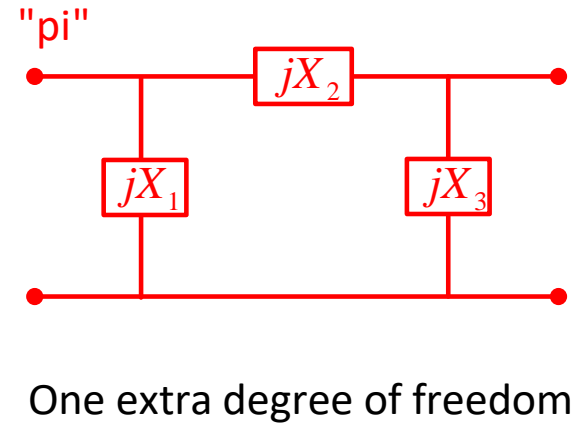
Lumped-Element Matching Circuits (cont.)

"tee"

jX_1 jX_3

jX_2

One extra degree of freedom

"ladder"

$m+2$ elements

m extra degrees of freedom

Impedance matching into complex termination

Let us consider what happens when the real parts of the source and load are the same, but one of them has a complex part, such as a series reactance as shown in the following figure.

Clearly, the presence of any net reactance between R_S and R_L reduces the current in R_L and with it the power dissipated in R_L. Therefore, to restore the dissipation to the maximum that occurs when $R_S = R_L$, the net reactance of the loop must be zero.

This occurs when the load and source are made to be complex conjugates of one another --- they have the same real parts and opposite type reactive parts.
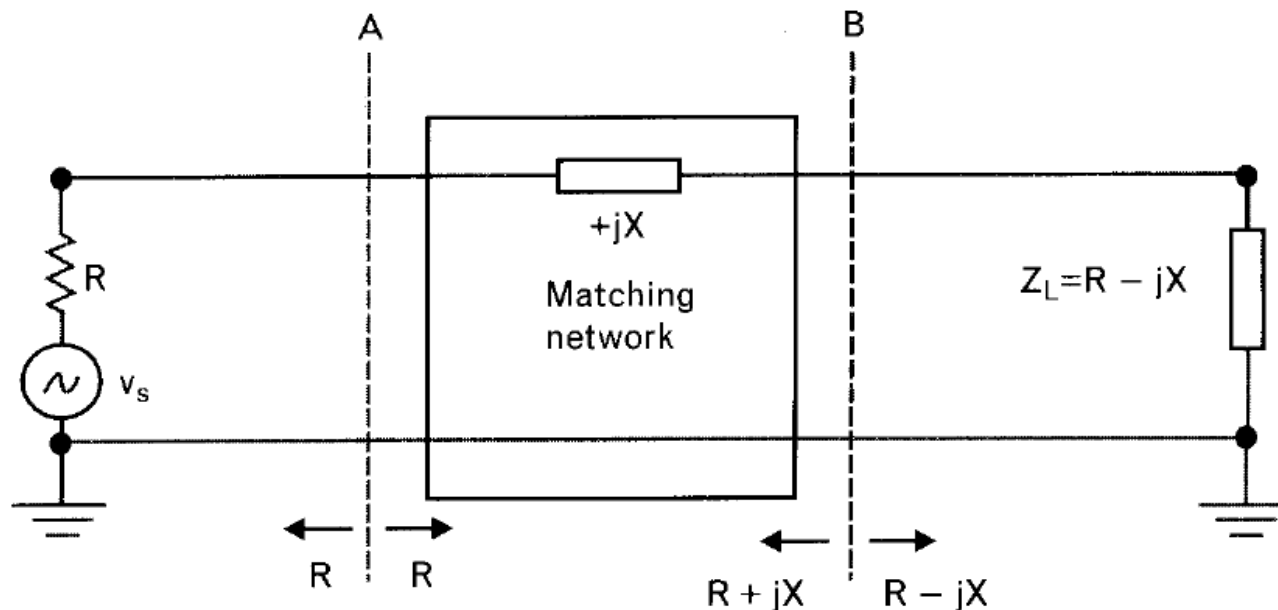

Figure: Impedance matching a resistive source and complex load for maximum power transfer. When the matching circuit is lossless, conjugate match exists at every interface.
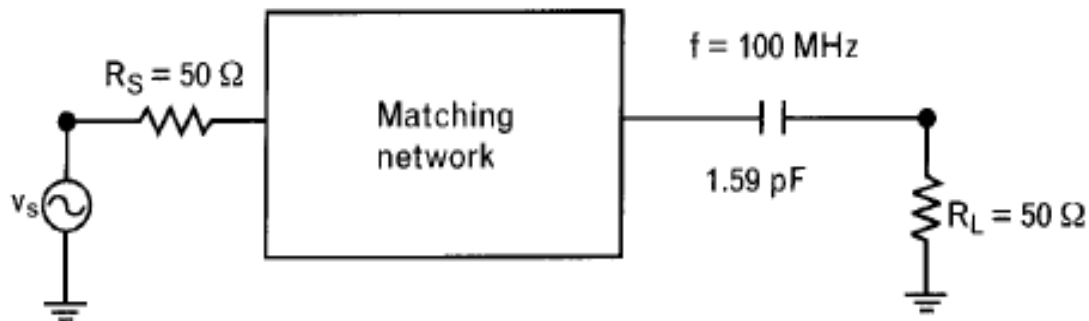
Match a 50-Ω resistive source at 100 MHz to a load that has an equivalent circuit of a 50-Ω resistor in series with a 1.59-pF capacitance, as shown in the following figure.

Solution

For the above circuit, we can easy get the matching network by a series inductor L:

$$j\omega L = -\frac{1}{j\omega C} \implies L = \frac{1}{\omega^2 C} = 1.59 \ \mu H$$

Conjugate match is obtained at only one frequency, f_R=100 MHz, where the two reactive elements resonate. As the frequency is increased or decreased from this value, the transmitted power rolls off at a rate determined by the loaded Q of the circuit as shown in the right figure.

Impedance matching technique

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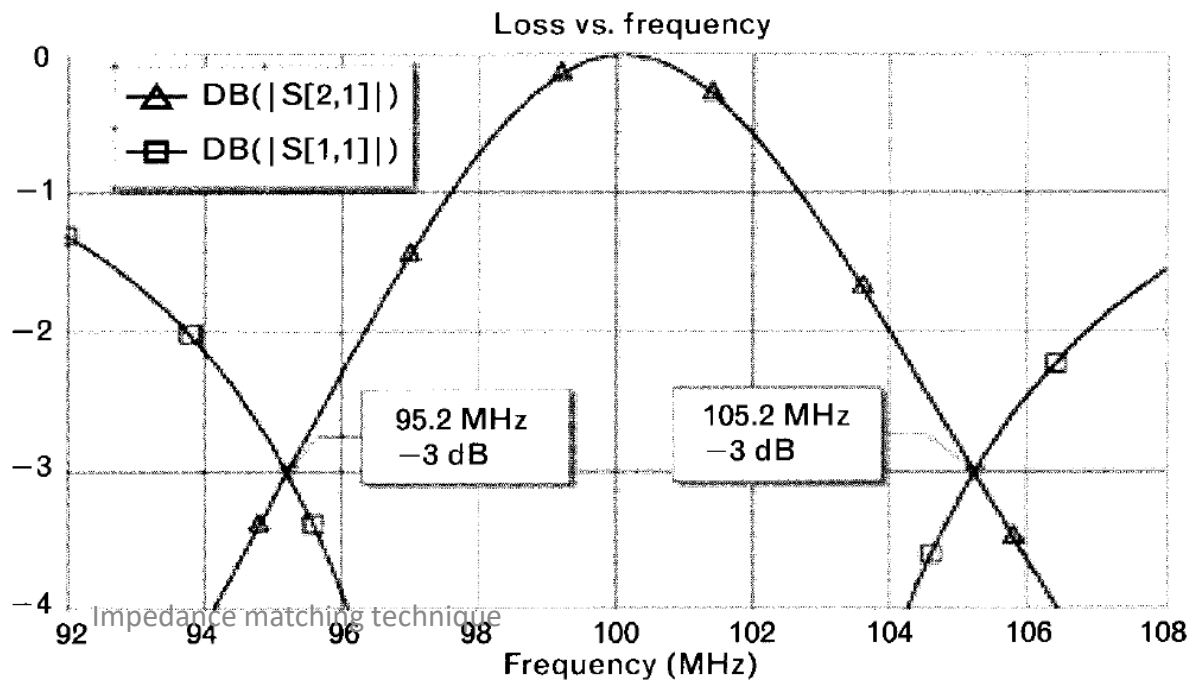
The 3-dB frequency bandwidth of a resonant matching network is found by:

$$BW_{3dB} = \frac{f_R}{Q_{SRES}}$$

Where Q_{SRES} is defined as

Loss vs. frequency

$Q_{SRES} = \dfrac{\text{Reactance of one resonant element at } f_R}{\text{Total series resistive loading}}$

For the parallel resonance, Q_{PRES}, is computed as

$Q_{PRES} = \dfrac{\text{Total parallel resistive loading}}{\text{Reactance of one resonant element at } f_R}$

Keep in mind that in parallel resonant circuits, high-value loading resistors lead to high Q, while in series circuits the opposite is true.

Impedance matching technique

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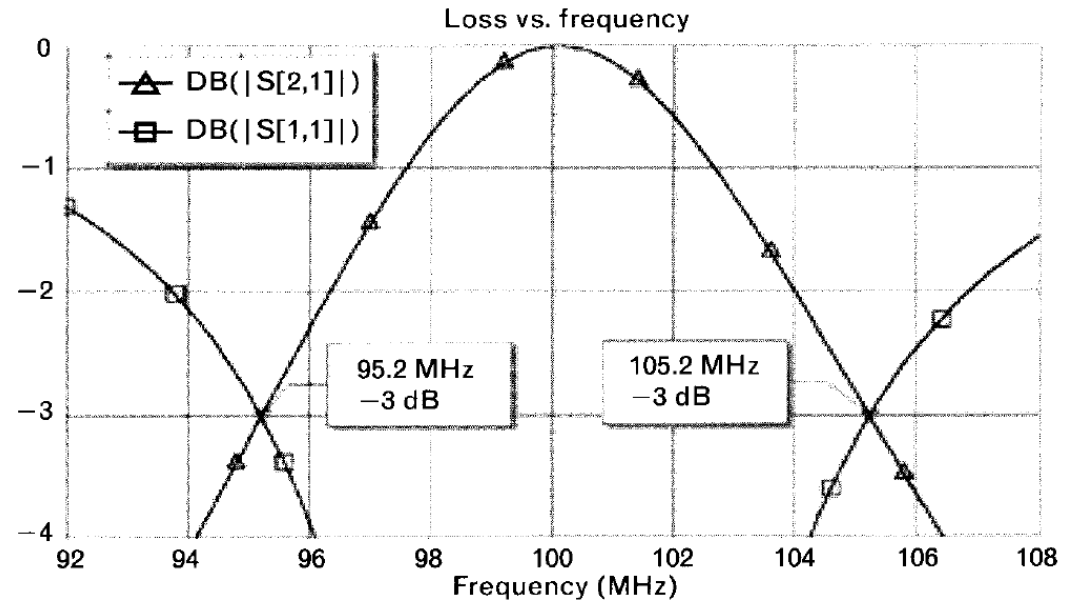
Impedance matching with uneven resistive terminations

In the previous example we had two terminations with equal real parts. Impedance matching in such cases is relatively easy. All we need to do is eliminate the effect of the reactive portion of terminations. In more practical applications, however, the resistive parts are not equal and the termination are complex.

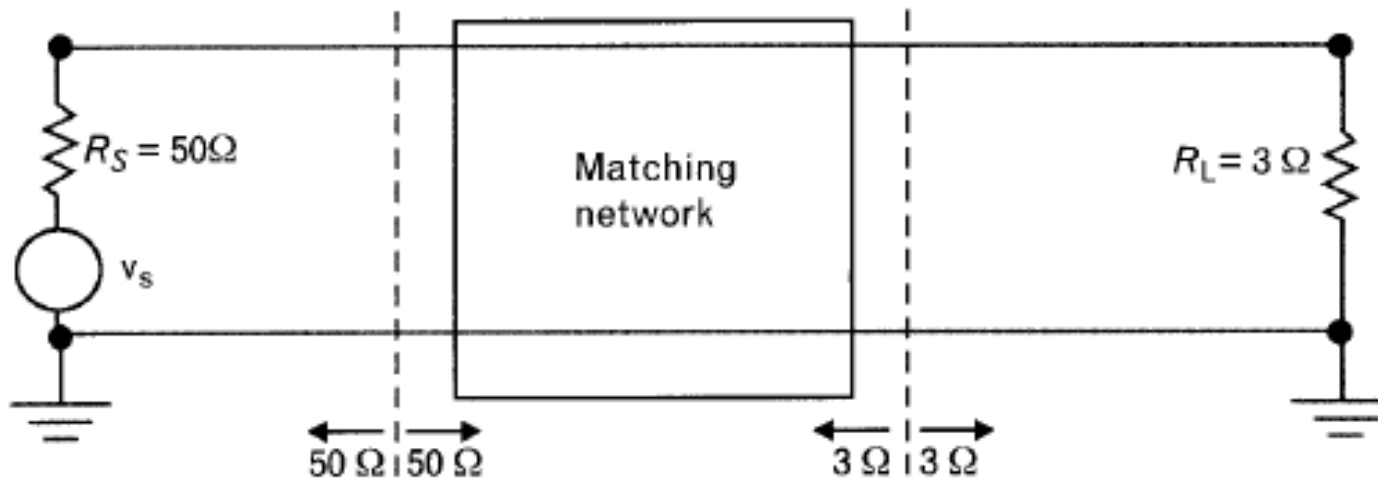## How to get a simple matching network described below?

Figure: How to get a simple matching network.

The Q matching technique with L-C sections

Any two resistive terminations can be simultaneously matched by adding two reactive elements between them if we follow four simple rules described below. For better visualization, we name the termination with lower resistance, $R_{SMALLER}$, and the other one R_{LARGER}.

1. Add a series reactive element next to $R_{SMALLER}$, and a parallel one to R_{LARGER}. We have the freedom of choosing whether the inductor or the capacitor is the series element, but the parallel element must to be the opposite type. If the series element is an inductor, adding a parallel capacitor creates a lowpass topology. A series capacitor with parallel inductor forms a highpass section.
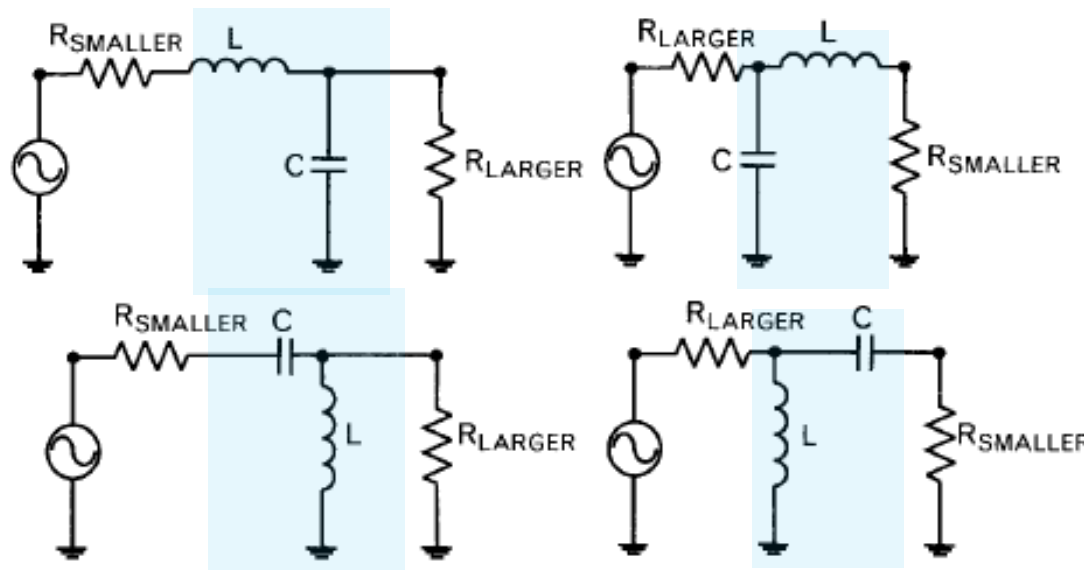
Figure: A single-section L-C circuit may be used to match a pair of arbitrary resistive terminations.

2. Calculating the reactance of the added reactive elements as two subnetworks. The Q-factors of these two subnetworks must be equal at the frequency where the match is computed. The Q-factors of the subnetworks, Q_S and Q_P are

$$Q_S = \frac{X_S}{R_{SMALLER}}, \qquad Q_P = \frac{R_{LARGER}}{X_P}$$

At the frequency where they are matched to each other, the two Q-factors are equal,
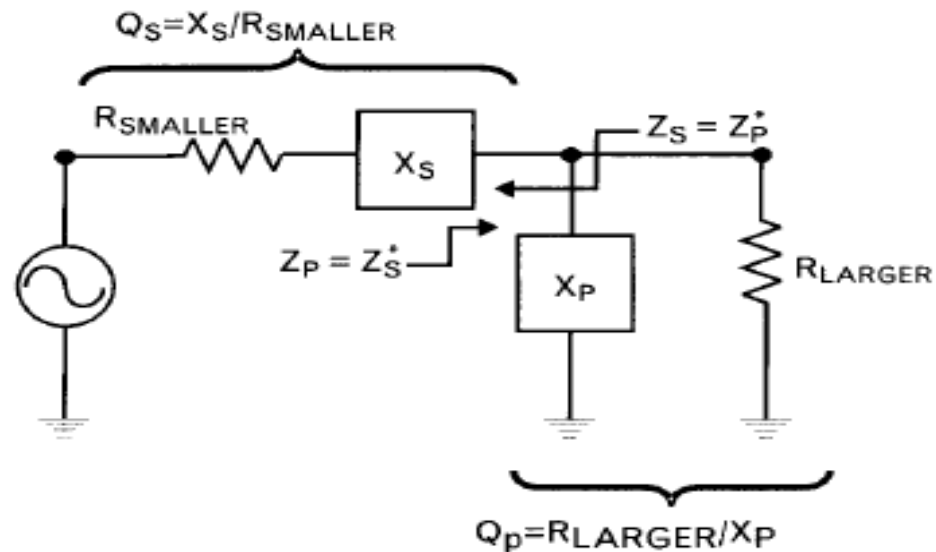
$$Q = Q_S = Q_P$$

Figure: An L-C section placed between the two resistive terminations creates a series and a parallel subnetwork.

Unloaded and loaded Q definitions

The unloaded quality-factor, Q_U, of a physical (nonideal) reactive component is defined as

$$Q_{Unload} = Q_U = \frac{\text{Energy stored in the component}}{\text{Energy dissipated in the component}}$$

When the component is used in a circuit, a loaded Q definition is useful:

$$Q_{Load} = Q_L = \frac{\text{Energy stored in the component}}{\text{Total energy dissipated in the component and external circuit}}$$
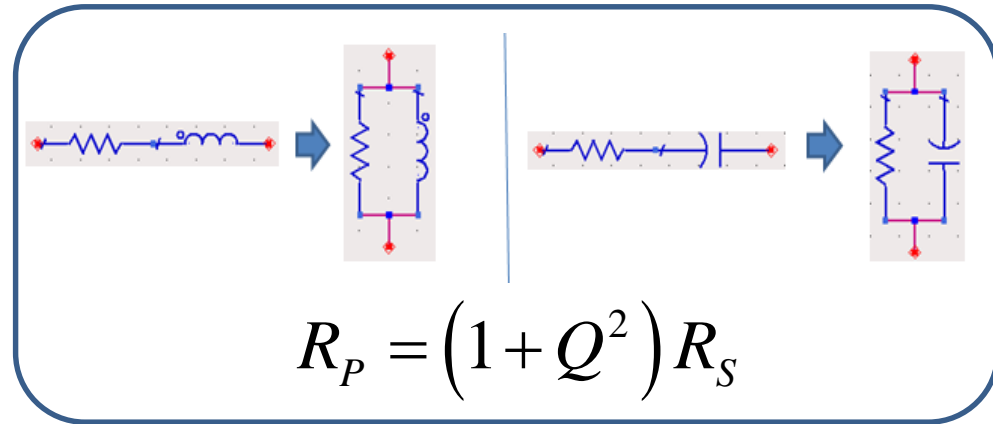

$$Q_S = \frac{X_S}{R_S}$$
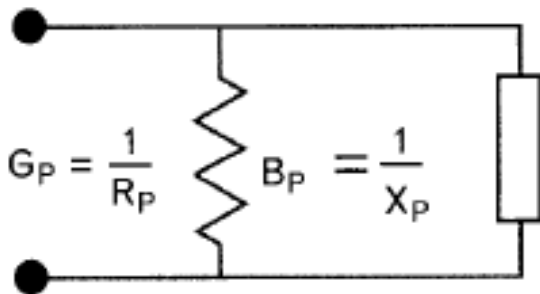
$$Q_P = \frac{B_P}{G_P} = \frac{1/X_P}{1/R_P} = \frac{R_P}{X_P}$$

Impedance matching technique

3. The Q-factor can be determined by

$$Q_S = Q_P = \sqrt{\frac{R_{LARGER}}{R_{SMALLER}} - 1}$$

$$R_P = \left(1 + Q^2\right) R_S$$

4. Knowing the Q values, we can obtain the series and parallel elements, reactances, and compute the inductor and capacitor values.

$$L = \frac{X_L}{\omega} = \frac{Q_S R_{SMALLER}}{\omega} = \frac{R_{LARGER}}{\omega Q_P}, \qquad C = \frac{1}{\omega X_C} = \frac{1}{\omega Q_S R_{SMALLER}} = \frac{Q_P}{\omega R_{LARGER}}$$

Example:

Using the Q-technique outlined, match a 5-Ω source to a 50-Ω resistive load at 850 MHz. Maintain dc connections between the two terminations.

Solution

Adding a series inductor to the 5-Ω side and a parallel capacitor to the 50-Ω side keeps the dc connection and creates the lowpass matching configuration as shown in the right figure.
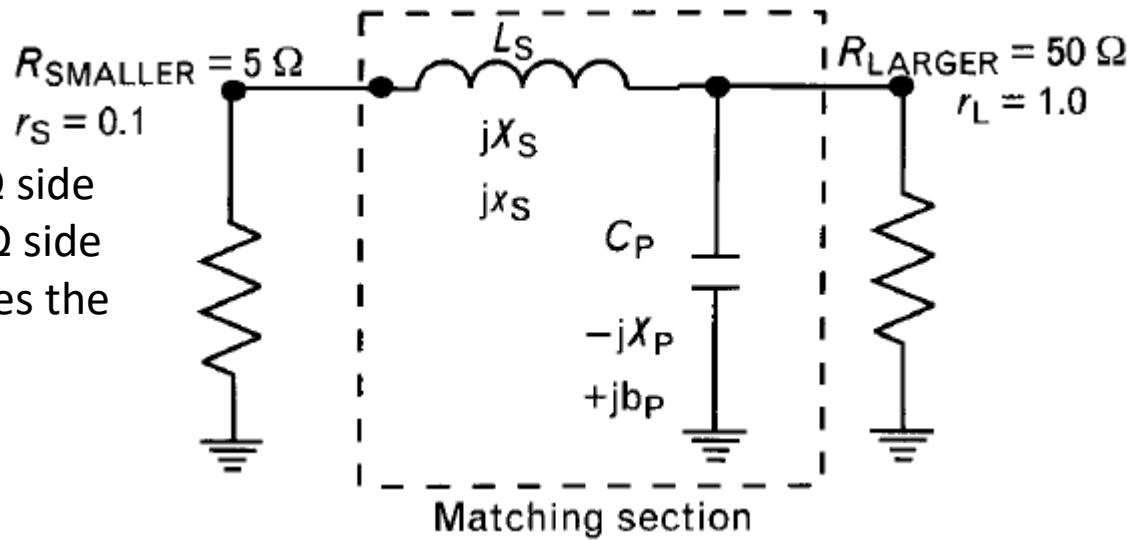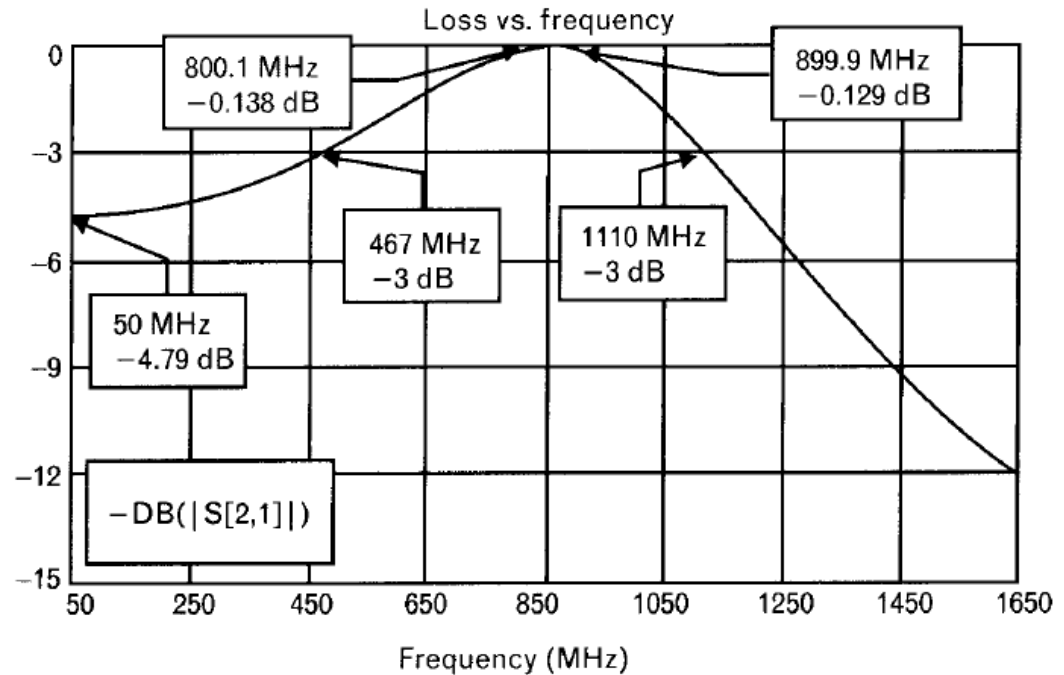
The Q-factors can be calculated as

$$Q_S = Q_P = \sqrt{\frac{R_{LARGER}}{R_{SMALLER}} - 1} = \sqrt{\frac{50}{5} - 1} = 3$$

Finding the inductor and capacitor values at 850 MHz from the above formulas:

$$L = \frac{Q_S R_{SMALL}}{\omega} =$$
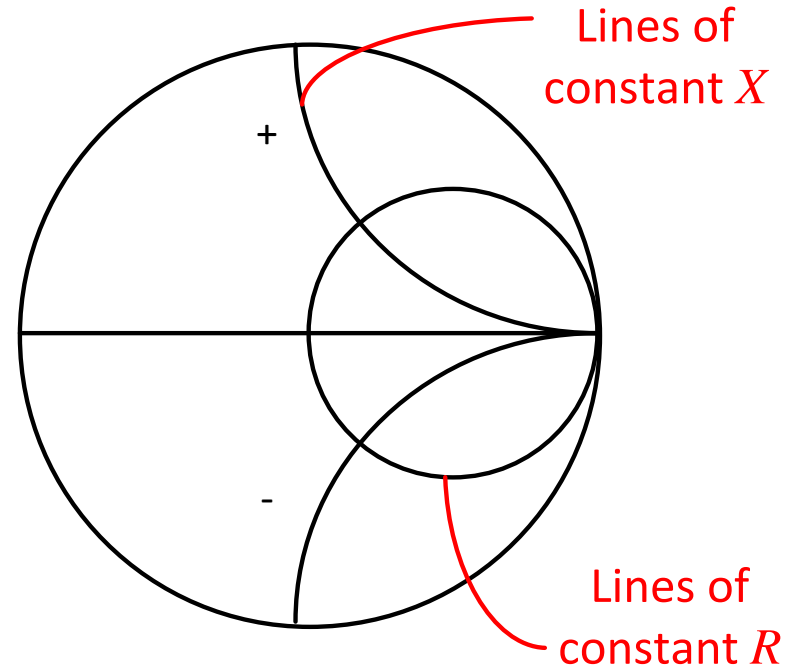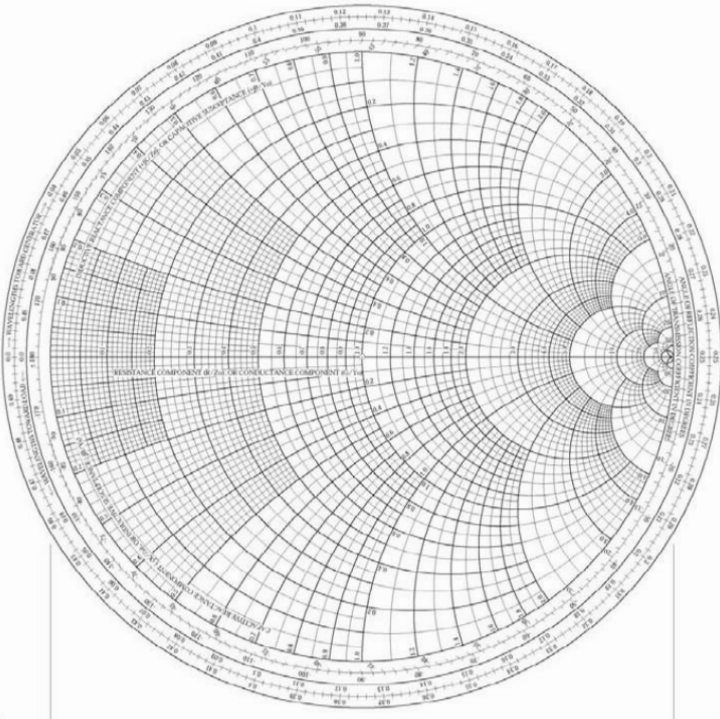
$$\frac{5*3}{2*\pi*850\times10^6} = 2.8\ nH$$

$$C = \frac{Q_P}{\omega R_{LARGER}} =$$

$$\frac{3}{2*\pi*850\times10^6*50} = 11.2\ pF$$

$R_{SMALLER} = 5\ \Omega$
$r_S = 0.1$

L_S

jX_S
jx_S

C_P

$-jX_P$
$+jb_P$

$R_{LARGER} = 50\ \Omega$
$r_L = 1.0$

Matching section

Loss vs. frequency

800.1 MHz
−0.138 dB

899.9 MHz
−0.129 dB

467 MHz
−3 dB

1110 MHz
−3 dB

50 MHz
−4.79 dB

−DB(|S[2,1]|)

Frequency (MHz)

Impedance matching technique

Smith Charts Review

Smith Chart
(Z-Chart)

Γ plane

Short-hand version

Lines of constant X

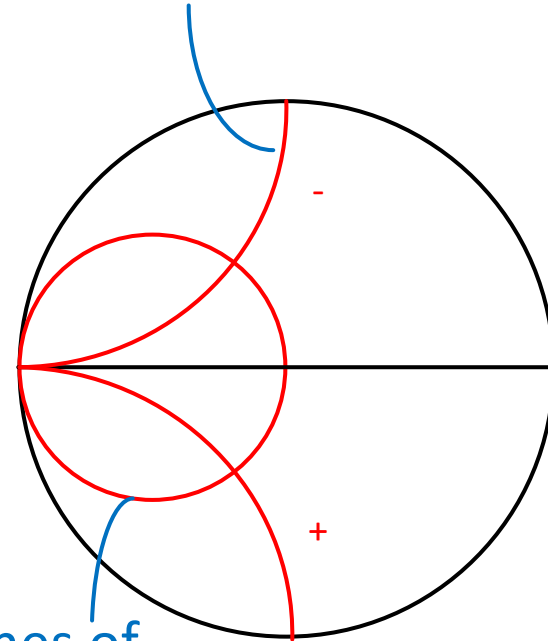Lines of constant R

+

-

Smith Charts Review (cont.)

Smith Chart
(Y-Chart)
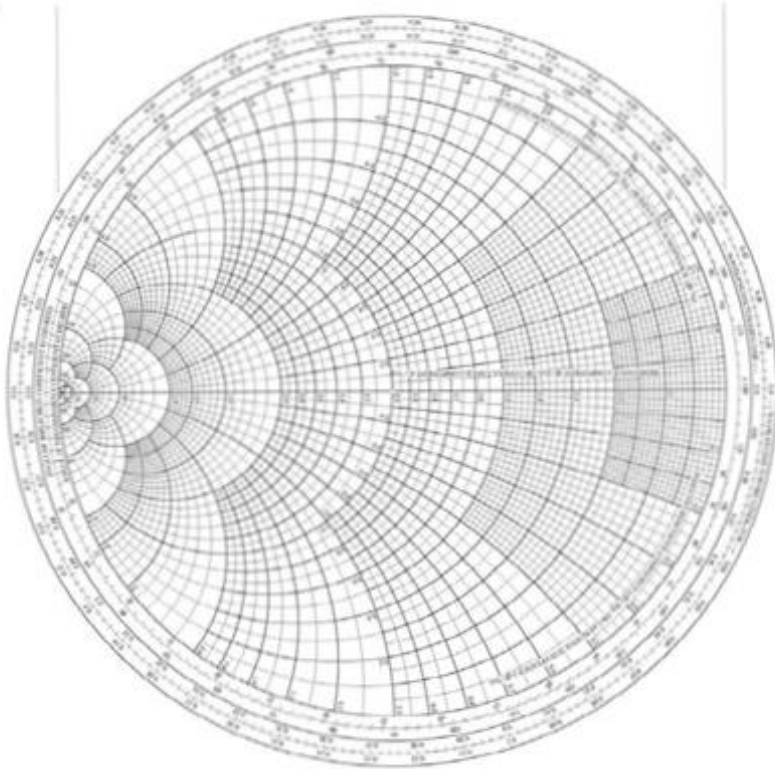
Γ plane

Lines of constant B
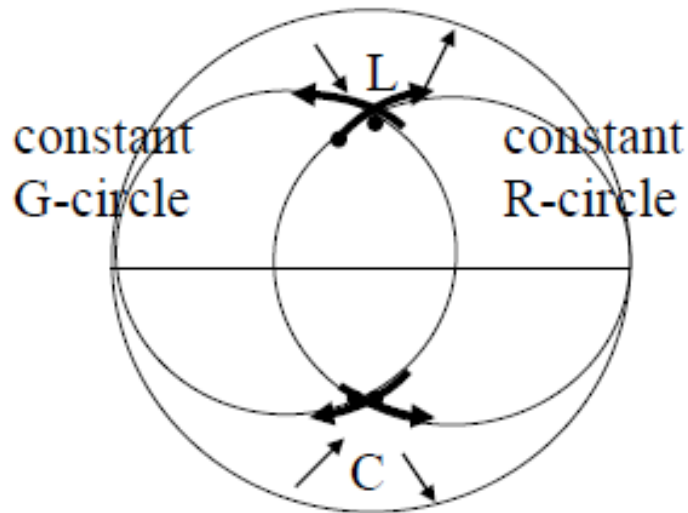
Lines of
constant G

-

+

Smith Charts Review (cont.)

ZY- Chart

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES
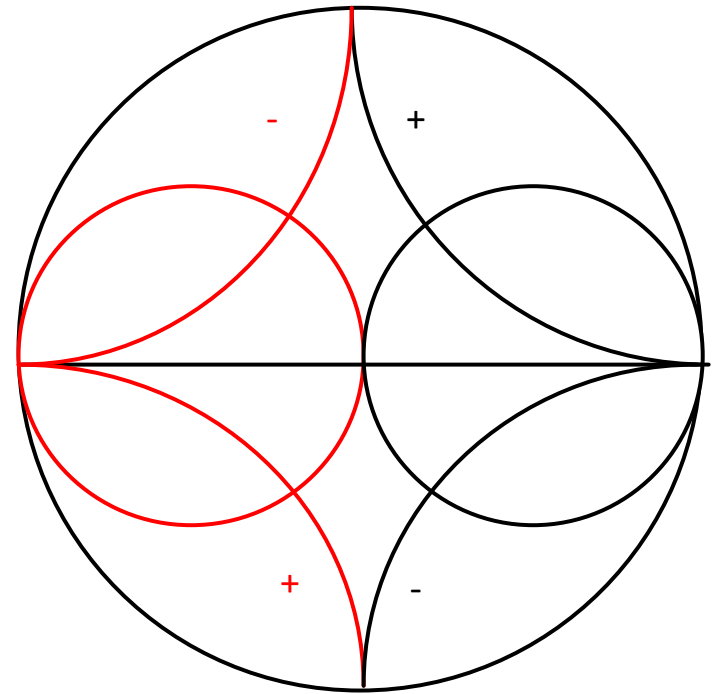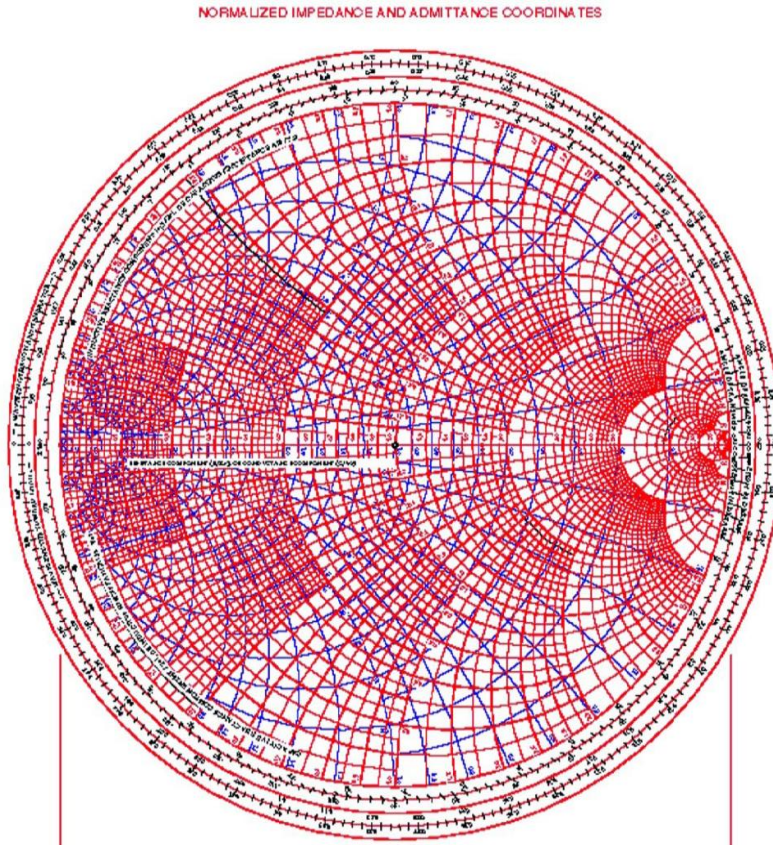
Γ plane

Series and Shunt Elements

• Smith chart solution

Z-plane

CW → add series L
    (reduce series C)
CCW → add series C
    (reduce series L)

Y-plane

CW → add shunt C
    (reduce shunt L)
CCW → add shunt L
    (reduce shunt C)

Series and Shunt Elements

Γ plane

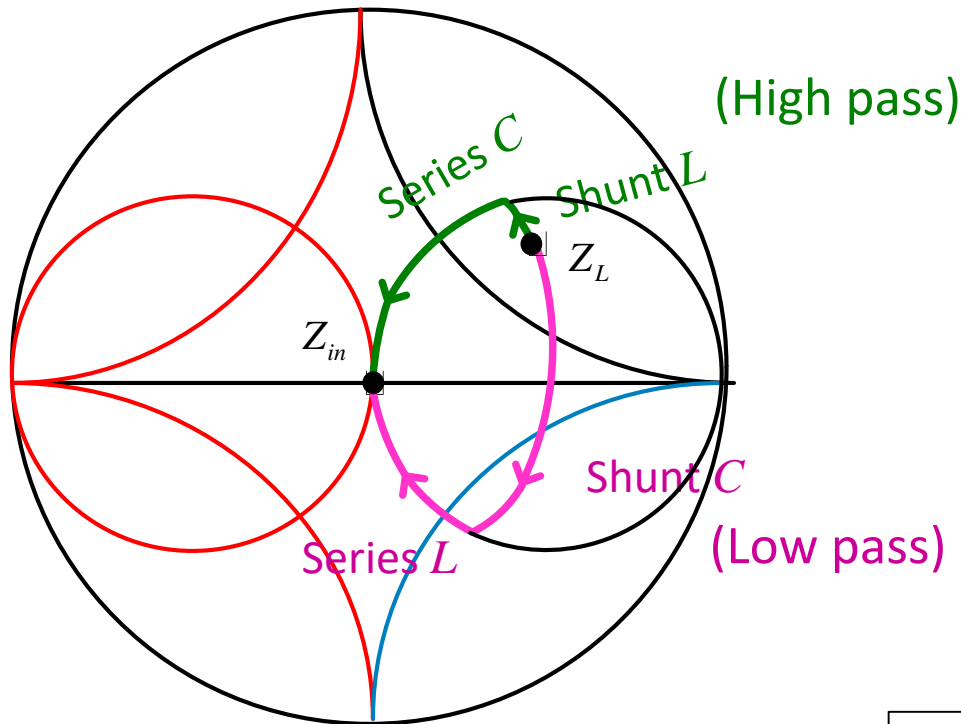Shunt L

Series L

Series C

Shunt C

Center of Smith chart:
Z_{in} (real)
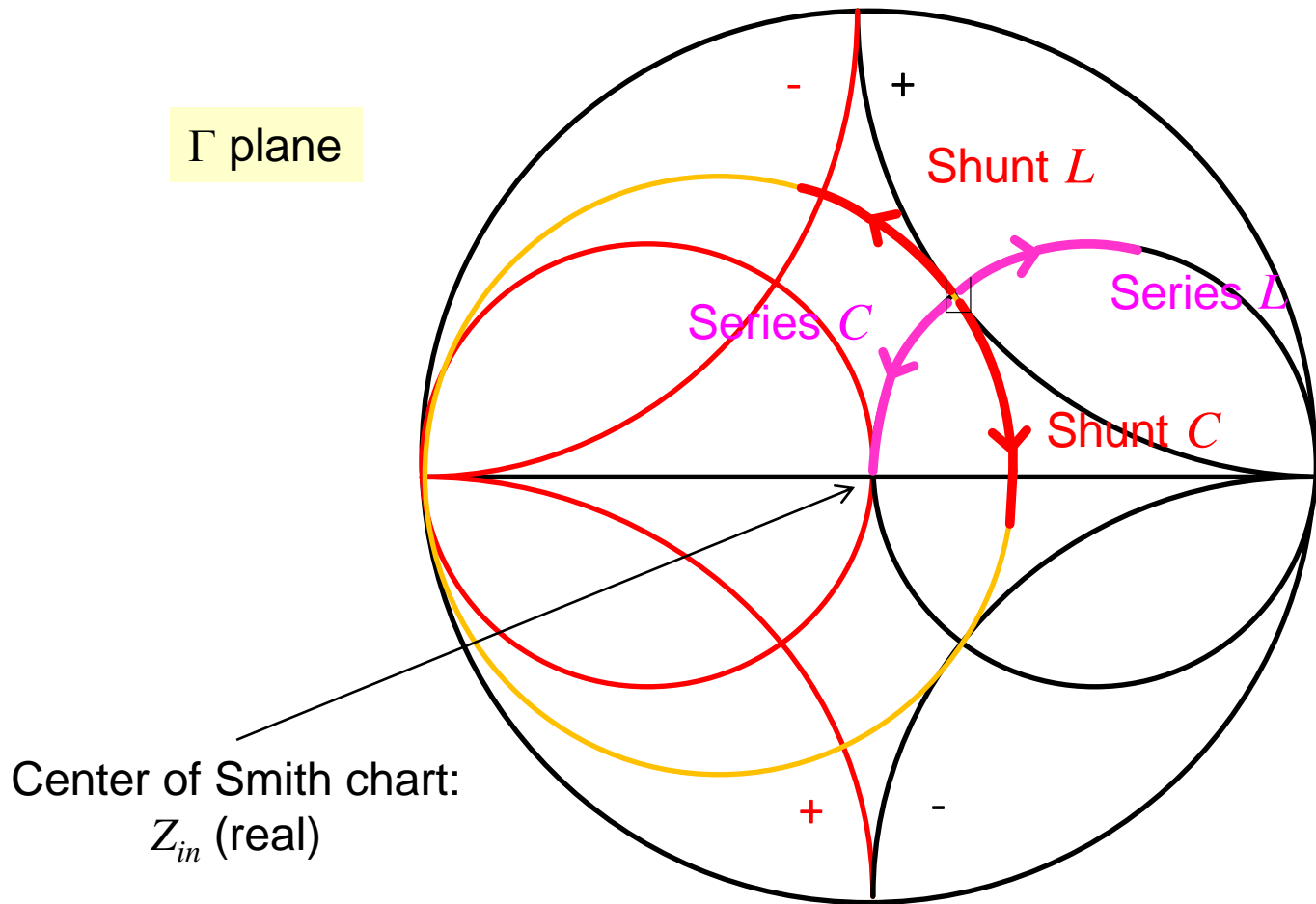
Note: The Smith chart is not actually being used as a transmission-line calculator but an impedance/admittance calculator. Hence, the normalizing impedance is arbitrary.
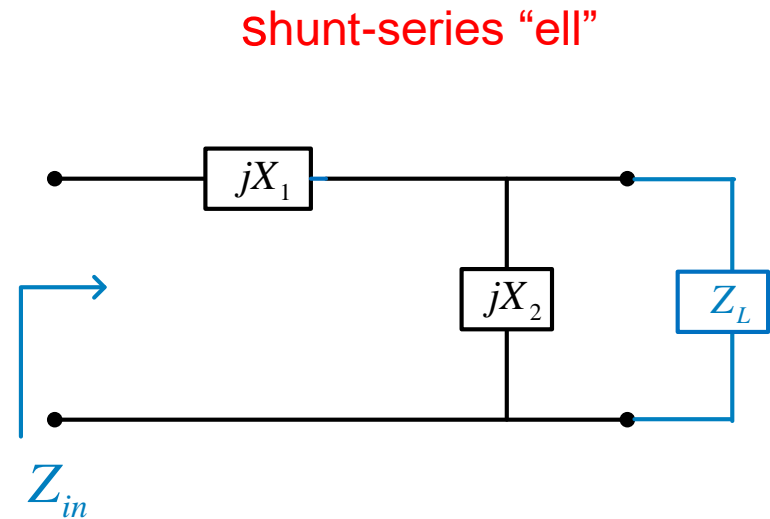
High Impedance to Low Impedance

Use when $G_L < Y_{in}$

(We are outside of the red $G = 1$ circle.)

(High pass)

Series C

Shunt L

Z_L

Z_{in}

Shunt C

Series L

(Low pass)

Shunt-series "ell"

jX_1

jX_2

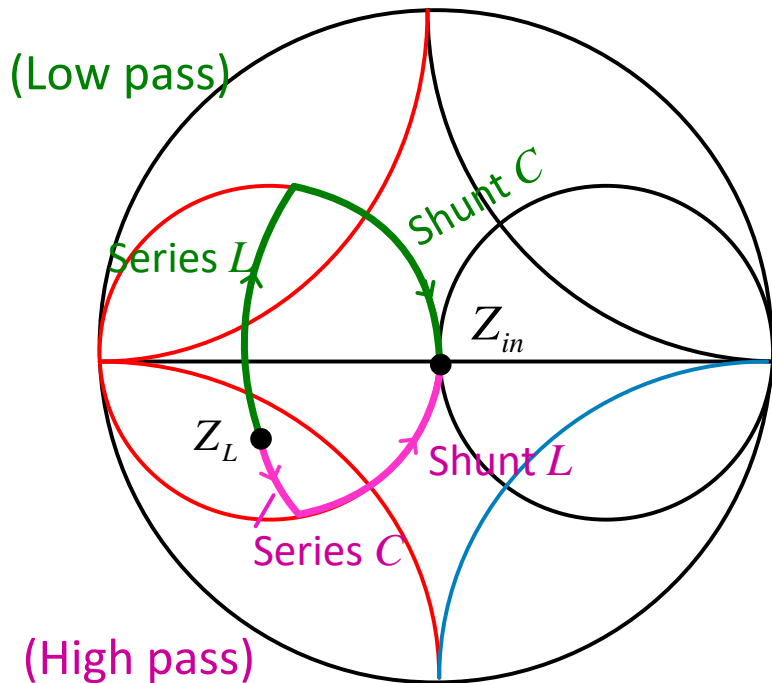Z_L

Z_{in}

Two possibilities

The shunt element decreases the impedance; the series element is used to "tune out" unwanted reactance.
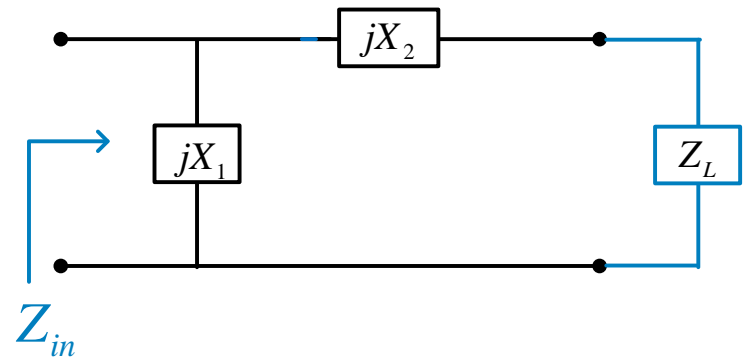
Low Impedance to High Impedance

Use when $R_L < R_{in}$

(We are outside of the black $R = 1$ circle.)

(Low pass)

Shunt C

Series L

Z_{in}

Z_L

Shunt L

Series C

(High pass)

Two possibilities

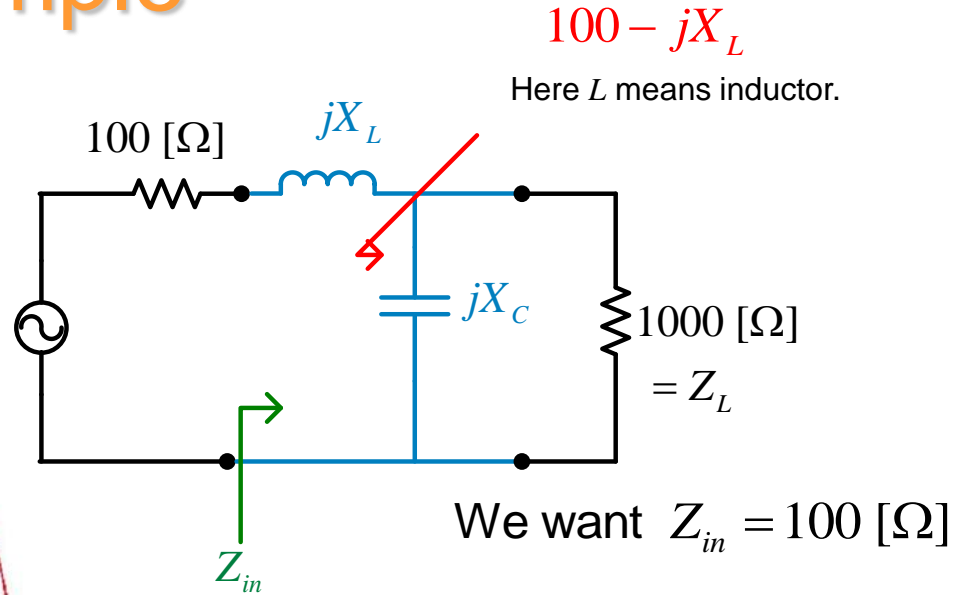Series-shunt "ell"
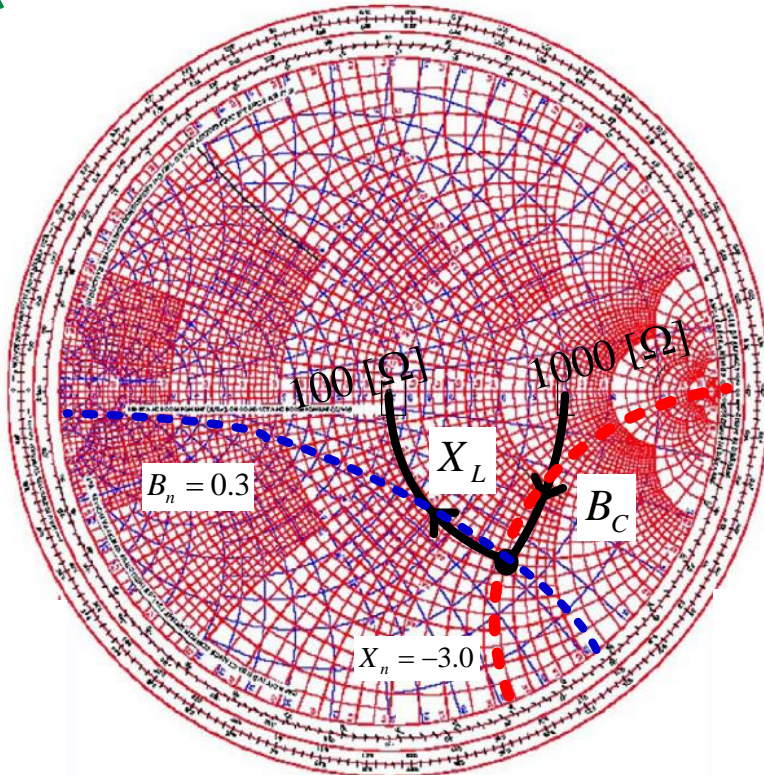
jX_2

jX_1

Z_L

Z_{in}

The series element increases the impedance; the shunt element is used to "tune out" the unwanted reactance.

Example

Choose normalizing impedance 100 [Ω]

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

$100 - jX_L$

Here L means inductor.

We want $Z_{in} = 100 \, [\Omega]$

$B_{C,n} = 0.3 \Rightarrow B_C = \dfrac{0.3}{100[\Omega]} = 3 \, [\text{mS}]$

$\Rightarrow X_C = -333[\Omega] = -\dfrac{1}{\omega C}$

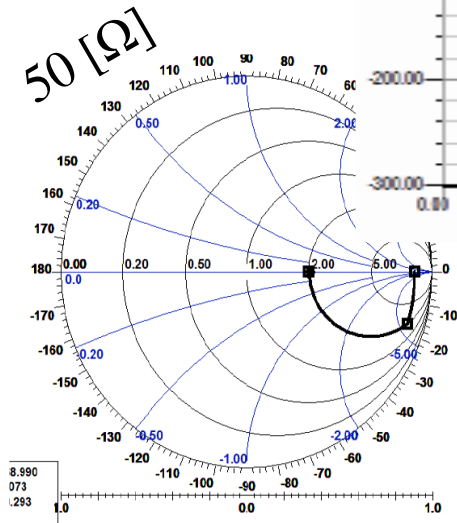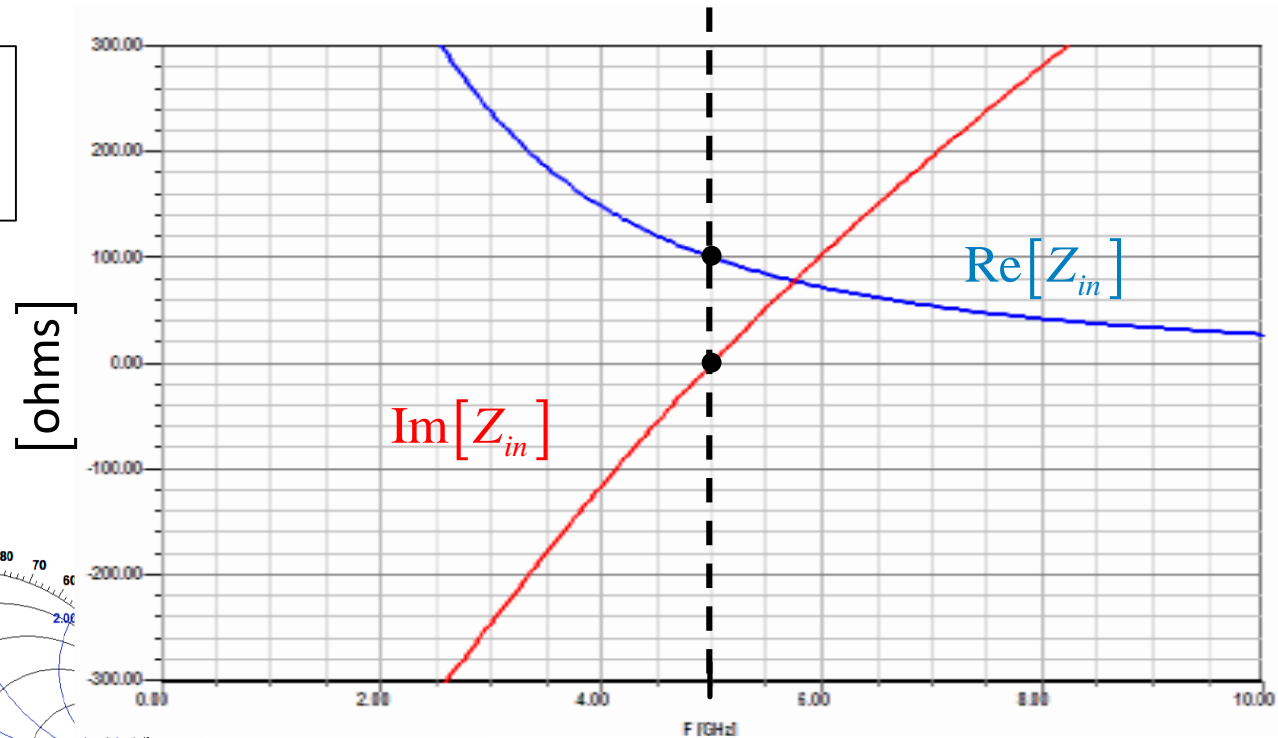$X_{L,n} = 3 \Rightarrow X_L = 3\left(100[\Omega]\right)$

$X_L = 300[\Omega] = \omega L$

Use low impedance to high impedance matching.

Example (cont.)

Here, the design example was repeated using a 50 [Ω] normalizing impedance.
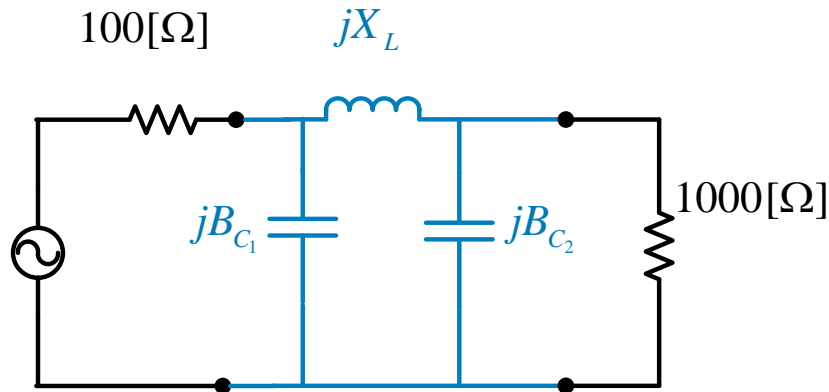
Note that the final normalized input impedance is 2.0.

$Re[Z_{in}]$

$Im[Z_{in}]$

[ohms]

50 [Ω]

5 GHz design frequency

$C = 0.096 \text{ [pF]}$

$L = 9.55 \text{ [nH]}$

Matching with a Pi Network

$100[\Omega]$ jX_L
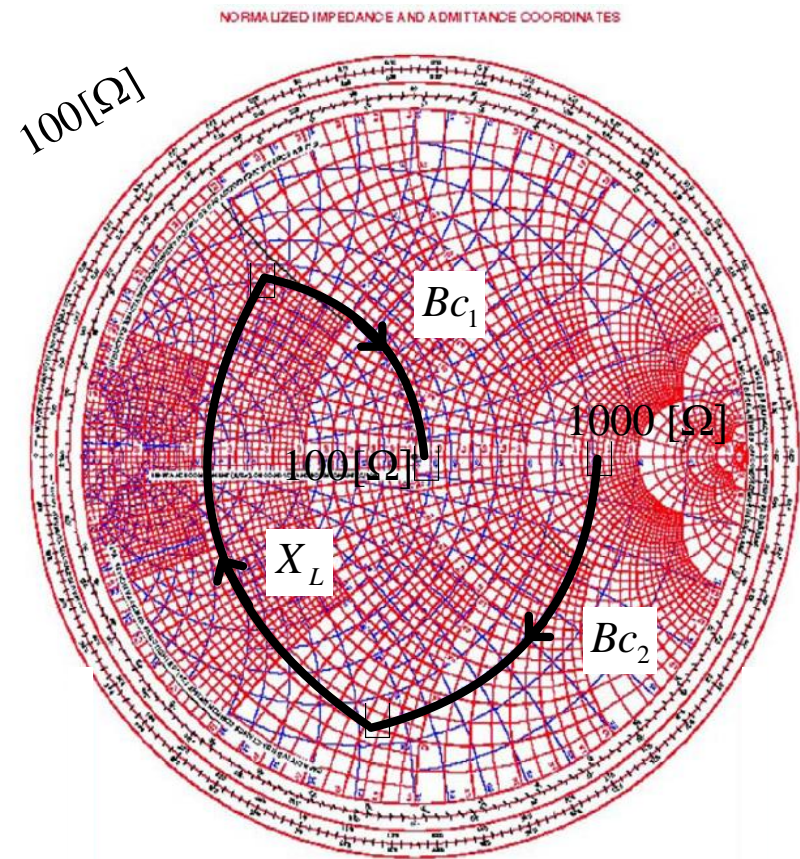
jB_{C_1} jB_{C_2} $1000[\Omega]$

This works for low-high or high-low.

The same example (1000 $\Omega \rightarrow$ 100 Ω) is used to illustrate matching with a pi network.

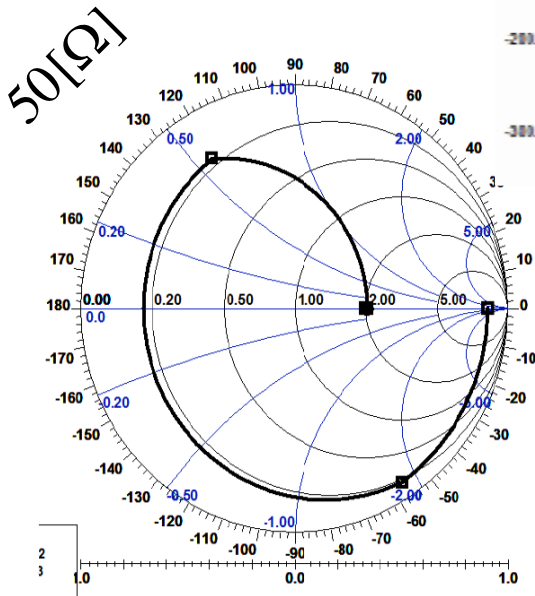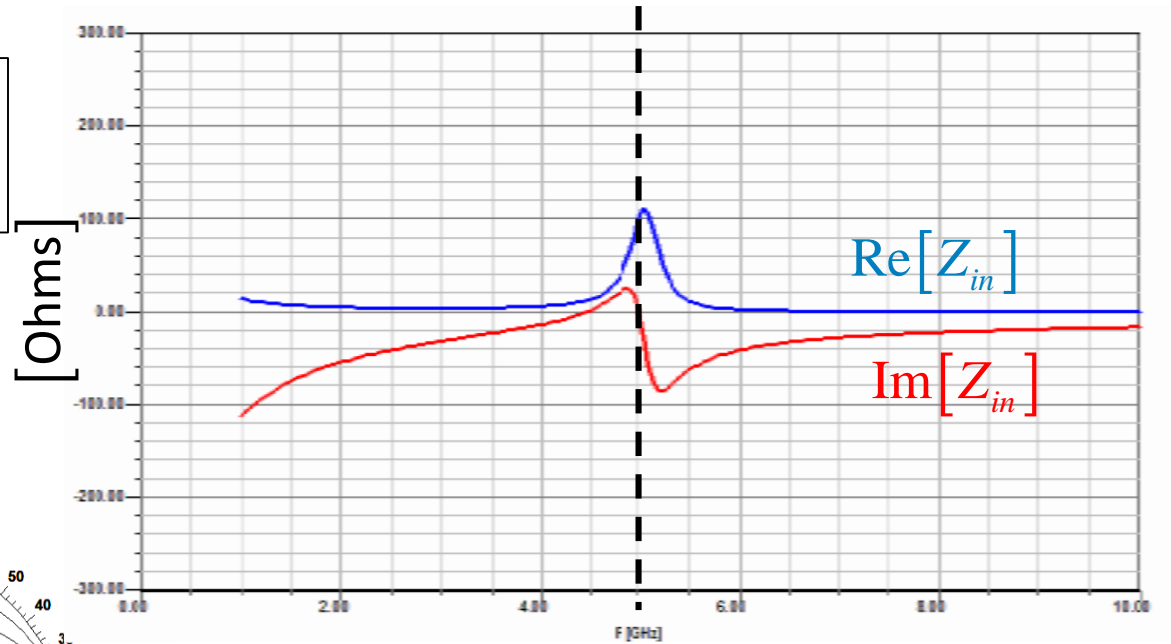Note that this solution is *not unique*. Different values for B_{c2} could have been chosen.

Note: We could have also used parallel inductors and a series capacitor, or other combinations.
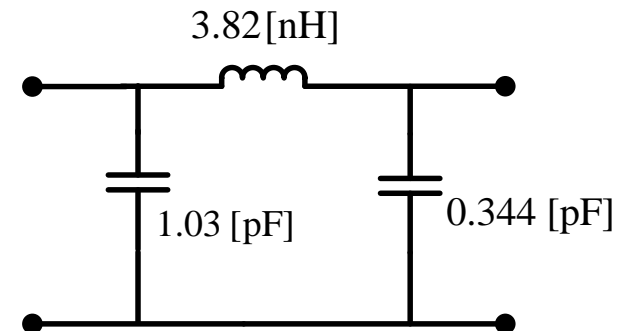
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

$100[\Omega]$

Bc_1

$1000 [\Omega]$

$100[\Omega]$

X_L

Bc_2

Pi Network Example

Note that the final normalized input impedance is 2.0.

$Re[Z_{in}]$

$Im[Z_{in}]$

[Ohms]

50[Ω]

Design frequency = 5GHz

3.82 [nH]

1.03 [pF]

0.344 [pF]

27

The end